4.3.2 EXERCISES

For a link to all of the additional resources available for this section, click OSttS Chapter 4 materials.

In Exercises 1 - 6, solve the rational equation. Be sure to check for extraneous solutions.

For help with these exercises, click one or more of the resources below:

- Solving equations involving rational expressions
- Identifying extraneous solutions

1. $\dfrac{x}{5x+4} = 3$
2. $\dfrac{3x-1}{x^2+1} = 1$
3. $\dfrac{1}{x+3} + \dfrac{1}{x-3} = \dfrac{x^2-3}{x^2-9}$
4. $\dfrac{2x+17}{x+1} = x+5$
5. $\dfrac{x^2-2x+1}{x^3+x^2-2x} = 1$
6. $\dfrac{-x^3+4x}{x^2-9} = 4x$

In Exercises 7 - 20, solve the rational inequality. Express your answer using interval notation.

For help with these exercises, click on the resource below:

- Solving inequalities involving rational expressions

7. $\dfrac{1}{x+2} \geq 0$
8. $\dfrac{x-3}{x+2} \leq 0$
9. $\dfrac{x}{x^2-1} > 0$
10. $\dfrac{4x}{x^2+4} \geq 0$
11. $\dfrac{x^2-x-12}{x^2+x-6} > 0$
12. $\dfrac{3x^2-5x-2}{x^2-9} < 0$
13. $\dfrac{x^3+2x^2+x}{x^2-x-2} \geq 0$
14. $\dfrac{x^2+5x+6}{x^2-1} > 0$
15. $\dfrac{3x-1}{x^2+1} \leq 1$
16. $\dfrac{2x+17}{x+1} > x+5$
17. $\dfrac{-x^3+4x}{x^2-9} \geq 4x$
18. $\dfrac{1}{x^2+1} < 0$
19. $\dfrac{x^4-4x^3+x^2-2x-15}{x^3-4x^2} \geq x$
20. $\dfrac{5x^3-12x^2+9x+10}{x^2-1} \geq 3x-1$

For help with Exercises 21 - 25, click Solving a work-rate problem.

21. Carl and Mike start a 3 mile race at the same time. If Mike ran the race at 6 miles per hour and finishes the race 10 minutes before Carl, how fast does Carl run?
22. One day, Donnie observes that the wind is blowing at 6 miles per hour. A unladen swallow nesting near Donnie's house flies three quarters of a mile down the road (in the direction of the wind), turns around, and returns exactly 4 minutes later. What is the airspeed of the unladen swallow? (Here, 'airspeed' is the speed that the swallow can fly in still air.)

23. In order to remove water from a flooded basement, two pumps, each rated at 40 gallons per minute, are used. After half an hour, the one pump burns out, and the second pump finishes removing the water half an hour later. How many gallons of water were removed from the basement?

24. A faucet can fill a sink in 5 minutes while a drain will empty the same sink in 8 minutes. If the faucet is turned on and the drain is left open, how long will it take to fill the sink?

25. Working together, Daniel and Donnie can clean the llama pen in 45 minutes. On his own, Daniel can clean the pen in an hour. How long does it take Donnie to clean the llama pen on his own?

26. In Exercise 32, the function $C(x) = .03x^3 - 4.5x^2 + 225x + 250$, for $x \geq 0$ was used to model the cost (in dollars) to produce x PortaBoy game systems. Using this cost function, find the number of PortaBoys which should be produced to minimize the average cost $\overline{C}$. Round your answer to the nearest number of systems.

27. Suppose we are in the same situation as Example 4.3.5. If the volume of the box is to be 500 cubic centimeters, use your calculator to find the dimensions of the box which minimize the surface area. What is the minimum surface area? Round your answers to two decimal places.

28. The box for the new Sasquatch-themed cereal, 'Crypt-Os', is to have a volume of 140 cubic inches. For aesthetic reasons, the height of the box needs to be 1.62 times the width of the base of the box.[13] Find the dimensions of the box which will minimize the surface area of the box. What is the minimum surface area? Round your answers to two decimal places.

29. Sally is Skippy's neighbor from Exercise 19 in Section 2.3. Sally also wants to plant a vegetable garden along the side of her home. She doesn't have any fencing, but wants to keep the size of the garden to 100 square feet. What are the dimensions of the garden which will minimize the amount of fencing she needs to buy? What is the minimum amount of fencing she needs to buy? Round your answers to the nearest foot. (Note: Since one side of the garden will border the house, Sally doesn't need fencing along that side.)

30. Another Classic Problem: A can is made in the shape of a right circular cylinder and is to hold one pint. (For dry goods, one pint is equal to 33.6 cubic inches.)[14]

    (a) Find an expression for the volume V of the can in terms of the height h and the base radius r.

    (b) Find an expression for the surface area S of the can in terms of the height h and the base radius r. (Hint: The top and bottom of the can are circles of radius r and the side of the can is really just a rectangle that has been bent into a cylinder.)

[13] 1.62 is a crude approximation of the so-called 'Golden Ratio' $\phi = \frac{1+\sqrt{5}}{2}$.

[14] According to www.dictionary.com, there are different values given for this conversion. We will stick with 33.6in^3 for this problem.

(c) Using the fact that $V = 33.6$, write S as a function of r and state its applied domain.

(d) Use your graphing calculator to find the dimensions of the can which has minimal surface area.

31. A right cylindrical drum is to hold 7.35 cubic feet of liquid. Find the dimensions (radius of the base and height) of the drum which would minimize the surface area. What is the minimum surface area? Round your answers to two decimal places.

32. In Exercise 71 in Section 1.4, the population of Sasquatch in Portage County was modeled by the function $P(t) = \frac{150t}{t+15}$, where $t = 0$ represents the year 1803. When were there fewer than 100 Sasquatch in Portage County?

In Exercises 33 - 38, translate the following into mathematical equations.

33. At a constant pressure, the temperature T of an ideal gas is directly proportional to its volume V. (This is Charles's Law)

34. The frequency of a wave f is inversely proportional to the wavelength of the wave λ.

35. The density d of a material is directly proportional to the mass of the object m and inversely proportional to its volume V.

36. The square of the orbital period of a planet P is directly proportional to the cube of the semi-major axis of its orbit a. (This is Kepler's Third Law of Planetary Motion)

37. The drag of an object traveling through a fluid D varies jointly with the density of the fluid ρ and the square of the velocity of the object ν.

38. Suppose two electric point charges, one with charge q and one with charge Q, are positioned r units apart. The electrostatic force F exerted on the charges varies directly with the product of the two charges and inversely with the square of the distance between the charges. (This is Coulomb's Law)

39. According to this webpage, the frequency f of a vibrating string is given by $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ where T is the tension, μ is the linear mass[15] of the string and L is the length of the vibrating part of the string. Express this relationship using the language of variation.

[15] Also known as the linear density. It is simply a measure of mass per unit length.

40. According to the Centers for Disease Control and Prevention www.cdc.gov, a person's Body Mass Index B is directly proportional to his weight W in pounds and inversely proportional to the square of his height h in inches.

    (a) Express this relationship as a mathematical equation.

    (b) If a person who was 5 feet, 10 inches tall weighed 235 pounds had a Body Mass Index of 33.7, what is the value of the constant of proportionality?

    (c) Rewrite the mathematical equation found in part 40a to include the value of the constant found in part 40b and then find your Body Mass Index.

41. We know that the circumference of a circle varies directly with its radius with 2π as the constant of proportionality. (That is, we know $C = 2\pi r$.) With the help of your classmates, compile a list of other basic geometric relationships which can be seen as variations.

Checkpoint Quiz 4.3

1. Solve: $\dfrac{1-x}{x^2-2x-1} \leq 0$

2. Solve: $\dfrac{4x}{x-1} \geq \dfrac{2x}{x+3}$

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2

4.3.3 ANSWERS

1. $x = -\frac{6}{7}$
2. $x = 1,\ x = 2$
3. $x = -1$
4. $x = -6,\ x = 2$
5. No solution
6. $x = 0,\ x = \pm 2\sqrt{2}$
7. $(-2, \infty)$
8. $(-2, 3]$
9. $(-1, 0) \cup (1, \infty)$
10. $[0, \infty)$
11. $(-\infty, -3) \cup (-3, 2) \cup (4, \infty)$
12. $\left(-3, -\frac{1}{3}\right) \cup (2, 3)$
13. $(-1, 0] \cup (2, \infty)$
14. $(-\infty, -3) \cup (-2, -1) \cup (1, \infty)$
15. $(-\infty, 1] \cup [2, \infty)$
16. $(-\infty, -6) \cup (-1, 2)$
17. $(-\infty, -3) \cup \left[-2\sqrt{2}, 0\right] \cup \left[2\sqrt{2}, 3\right)$
18. No solution
19. $[-3, 0) \cup (0, 4) \cup [5, \infty)$
20. $\left(-1, -\frac{1}{2}\right] \cup (1, \infty)$
21. 4.5 miles per hour
22. 24 miles per hour
23. 3600 gallons
24. $\frac{40}{3} \approx 13.33$ minutes
25. 3 hours
26. The absolute minimum of $y = \overline{C}(x)$ occurs at $\approx (75.73, 59.57)$. Since x represents the number of game systems, we check $\overline{C}(75) \approx 59.58$ and $\overline{C}(76) \approx 59.57$. Hence, to minimize the average cost, 76 systems should be produced at an average cost of \$59.57 per system.
27. The width (and depth) should be 10.00 centimeters, the height should be 5.00 centimeters. The minimum surface area is 300.00 square centimeters.
28. The width of the base of the box should be ≈ 4.12 inches, the height of the box should be ≈ 6.67 inches, and the depth of the base of the box should be ≈ 5.09 inches; minimum surface area ≈ 164.91 square inches.
29. The dimensions are ≈ 7 feet by ≈ 14 feet; minimum amount of fencing required ≈ 28 feet.
30. (a) $V = \pi r^2 h$
    (b) $S = 2\pi r^2 + 2\pi r h$
    (c) $S(r) = 2\pi r^2 + \frac{67.2}{r}$, Domain $r > 0$
    (d) $r \approx 1.749$ in. and $h \approx 3.498$ in.
31. The radius of the drum should be ≈ 1.05 feet and the height of the drum should be ≈ 2.12 feet. The minimum surface area of the drum is ≈ 20.93 cubic feet.
32. $P(t) < 100$ on $(-15, 30)$, and the portion of this which lies in the applied domain is $[0, 30)$. Since $t = 0$ corresponds to the year 1803, from 1803 through the end of 1832, there were fewer than 100 Sasquatch in Portage County.

33. $T = kV$

34. [16] $f = \dfrac{k}{\lambda}$

35. $d = \dfrac{km}{V}$

36. $P^2 = ka^3$

37. [17] $D = k\rho\nu^2$

38. [18] $F = \dfrac{kqQ}{r^2}$

39. Rewriting $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$ as $f = \dfrac{\frac{1}{2}\sqrt{T}}{L\sqrt{\mu}}$ we see that the frequency f varies directly with the square root of the tension and varies inversely with the length and the square root of the linear mass.

40. (a) $B = \dfrac{kW}{h^2}$ (b) [19] $k = 702.68$ (c) $B = \dfrac{702.68W}{h^2}$

[16] The character λ is the lower case Greek letter 'lambda.'
[17] The characters ρ and ν are the lower case Greek letters 'rho' and 'nu,' respectively.
[18] Note the similarity to this formula and Newton's Law of Universal Gravitation as discussed in Example 5.
[19] The CDC uses 703.